Quantifying Temperature and Barometric Effects in Free Pendulum Clocks

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We seem to find it difficult to relate theoretical predictions of clock performance to actual clocks. I started my 7th clock on June 20, 1998 after discussing with friends various mysteries from my previous 33 years and 6 clocks. They ask why it wouldn't be possible to make a 120 beat clock as good at the one on Grandma's mantel.

My interest is in free pendulum clocks and all have used electromagnetic drive. Various optical sense systems have been used. Figure 1 shows the pendulum configuration. Two infrared switches are used. One indicates left/right of center and the other bounds the swing to about 1.5 degrees (half angle.) This arrangement keeps circular error very nearly constant. The pendulum is supported in a wooden frame and sits on my lab bench on the second floor of a frame house with forced air heating. Our house is less than 10 miles from the San Andreas fault. I do get some amount of small earthquake shock in the data stream. This is not designed to be a perfect pendulum but rather a platform for experiments to try to characterize temperature and pressure effects.

I think that an equation such as

$$P = aT + bB + c$$

where P is the period of the pendulum, a is a coefficient of temperature, b is a coefficient of barometric pressure, and c is some constant should fit the performance of an actual clock. Coefficient 'a' can capture all the various temperature effects and likewise 'b' can capture everything related to pressure. In a house the temperature is likely limited to a 15°C range and barometric pressure changes won't be greater than about 1 inHg so this equation may well be sufficient. If this is the case a microcontroller chip can easily remove rate error from a simple clock. If not I can increase the complexity of the equation until accuracy of 1 ppm or perhaps 0.1 ppm is achieved without resorting to vacuum chambers and separate buildings. This, at least, is my hope.

In reviewing all the HSC newsletters, I didn't find complete documentation of any optical sense system although many systems seem to be in use. Figure 2 shows the complete electronics and a timing diagram relating the various signals to the position of the pendulum. The 'drive' basically notices when the right most displacement of the pendulum doesn't quite reach the limit sensor and pulses the electromagnet for 8ms just at the bottom of the swing. Schmitt trigger input inverters are used to interface the infrared switches. The logic used is 74HC although anything will do. This circuit doesn't depend on any time source other than the pendulum itself. In operation 1 pulse is added in 12 cycles. I have not measured Q. I have used infrared pairs for position sensing for more than 10 years in many configurations and have found them to be accurate and trouble free.

The 1 Hz 'tick' signal is routed to a PIC (16F84) microcontroller chip operating at 10MHz. Figure 3 shows a high level schematic. This system is not unlike those previously documented (Thackery HSC 1998-2 and Mumford HSC 1997-4.) The system has a timing resolution of 0.1 µsec; however, results are reported in 52 µsec bins. This system includes temperature sensing with a resolution of 0.01 °C and ±0.5 °C absolute accuracy from a single Dallas DS1620 chip. Barometer data is also provided with a resolution of better than 0.01 inHg and absolute accuracy of about 0.1 inHg. These transducers are done inexpensively using the PIC and Linux for all the hard work. If there is interest, I will make a detailed description of both the electronics and the calibration procedure in a future article.

Timing, temperature, and barometric information are routed through a RS232 serial interface and gathered by a freely available Unix system called Linux. I've written a number of programs to process the raw data and produce input for a plotting package called IPL. The PIC produces about 1Mbyte of data every other day so input files are quite large. Individual programs are used to isolate data that is gathered on a cycle by cycle basis. This means I can look at every cycle to a resolution of 52 µsec for months at a time.

Figure 4 shows about 1M seconds of operation starting about the first of 1999. 86400 samples are averaged to produce results by day. The house thermostat does a good job, on a day basis, keeping constant temperature (±0.5 °C.) The barometer was busy for the entire period of 11 days. Period is shown and ranges only ±1 µsec. The strip shown is corrected for temperature (in addition to the aluminum tube on the pendulum) by 1.8 ppm/°C and for barometric pressure changes by 15 ppm/inHg. The leveled rate is shown in the bottom strip and drifts slow by

180 msecs in the second half of the chart. Detailed notes for this plot are:

total samples:	950400
mean period:	999.760118 ms
period range:	0.002300 ms
mean temp:	21.295 °C
mean baro:	30.22 inHg
temp correction:	1.8 ppm/°C
baro correction:	15.0 ppm/inHg

(This is not at all certain. I have seen values from 5 to 20 ppm/inHg.)

PIC reports time intervals in bins of 52 μsec but the bins are directly related to the 10MHz crystal and so have a resolution of 0.1 μsec. Below is a histogram showing the number of cycles falling in bins near the mean:

Bin	Number of cycles	
	124 in shorter bins	
19221	32	
19222	54	
19223	103	
19225	38196	999.700 ms
19226	725358	999.752 ms
19227	184801	999.804 ms
19228	1077	
19229	122	
19230	51	
19231	30	
	148 in longer bins	

(eg. 19226 X 52 μsec = 999.752 ms)

One might expect that all samples would fall in bin 19226 (the one smaller than the mean) or 19227 (the one larger than the mean) in proportion to the mean. This is not the case and a good deal of jitter is seen in practice. The program which produces the histogram above shows all outliers and when they occur. Many are shocks lasting less than 5 seconds with bins below the mean balanced by bins above the mean. Experience also shows that most are during the times I am moving about in my lab.

For those of us who have happily rated our clocks by hand each morning or night Figure 5 will be something of shock. Here are hour averages for the 5th and 6th day of Figure 4. Raw period in the center tracks temperature at 1.71 ppm/°C with a correlation coefficient of 87%. Correcting at this rate gives the (tc) period which correlates with the derivative of temperature (71%.) Clearly the quick changes of temperature seen as the house cools off at night and reheats starting at 5:30AM are large. Peak to peak changes of 5 ppm in 2.5°C are seen in this plot even after the 1.71 correction. For short time intervals my equation above is too simple and a dT/dt term is needed.

Perhaps most troubling is the fact that after all temperature compensations are applied to the data what remains is almost completely uncorrelated to barometer (5%.)

Many have reported that clocks run fine and then abruptly change rate. I find the same thing and can actually catch my clock doing it. Figure 6 shows a rate change of nearly 20 ppm over a 14 hour span. No clues are found in the either temperature or barometer (the apparent barometer negative correlation is 123 ppm/inHg (54%) is so large that it would have shown up in all the other plots. This, I think, is just a trick of plotting scale and chance.) This 20 ppm shift takes place between 16 and 28 hours. Before 16 hours the normal 1.71 ppm/°C rate holds. After 28 hours the normal rate holds briefly then the period shortens for the next 7 days and 10 ppm less than the previous average period. Pendulum periods more than ±3 bins from the mean are shown in the bottom strip. The 2.8 msec burst is about 56 bins from the center and is in fact a very rare event (there are only about 4 such events in 60 days.) It occurs more than 6 hours before the period begins to change and there is no evidence of any effect near the burst. Note also the nearly matching negative outlier. A 20 ppm change is about 10 microns of effective pendulum length or 0.01 mm. Furthermore period changes smoothly through the entire 12-hour event.

Figure 7 tabulates my experience with various rods, flexures, and compensating tubes. The measured residual temperature coefficient is shown and the theoretical performance of the rod, compensator, and bob is computed. In the 7/8/98 tests vs. the 8/11 tests we see a large effect likely from the tempco of the flexure. In the current tests it seems likely that the tempco of my raw invar is a good deal greater than 1.4e-6. There is more uncertainty in earlier tests than current ones. Current tempco is likely to be within ±10%, but if anything this paper shows that this is not an easy measurement to make.

Figure 8 shows more than 60 days of operation and many unexplained shifts in period. My current experimental setup goes part of the way to my goal of quantifying temperature and barometric effects; however, instabilities both large and small still greatly hamper this work. Comments are most welcome.

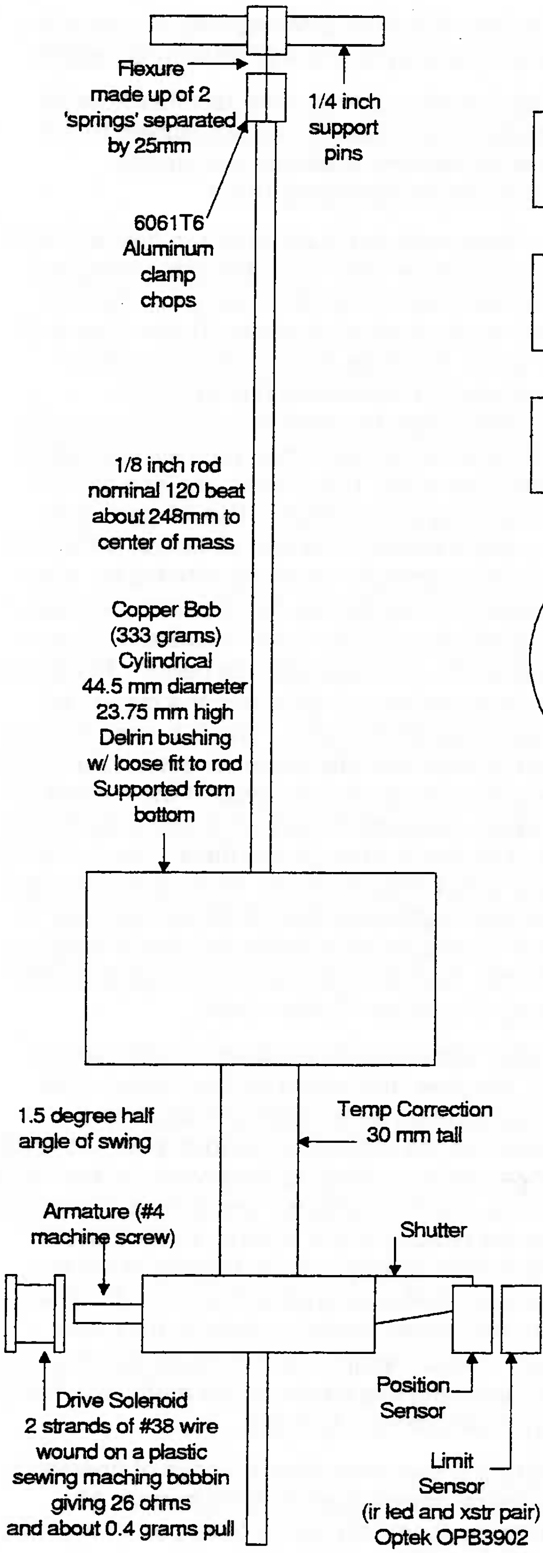


Figure 1. C7 Pendulum front view (not to scale)

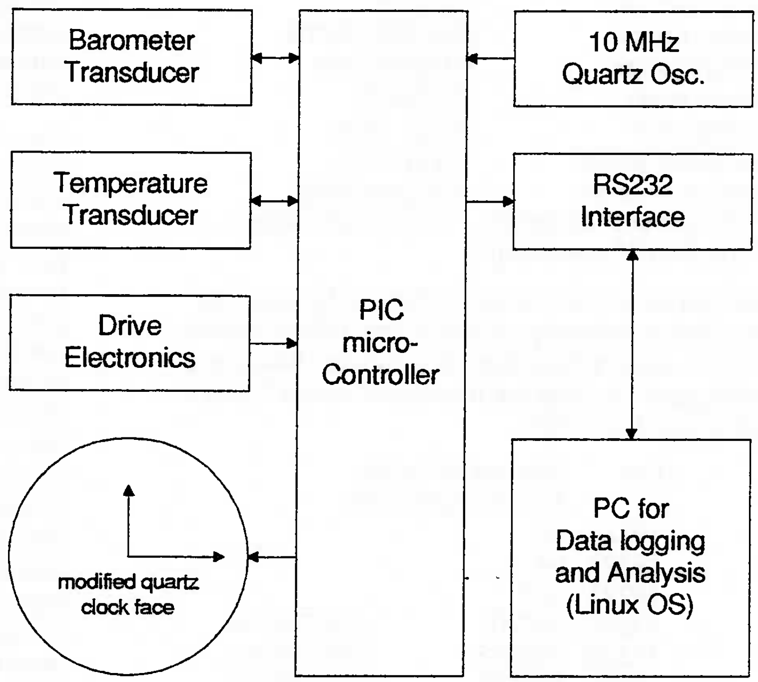


Figure 3. Data Capture and Analysis

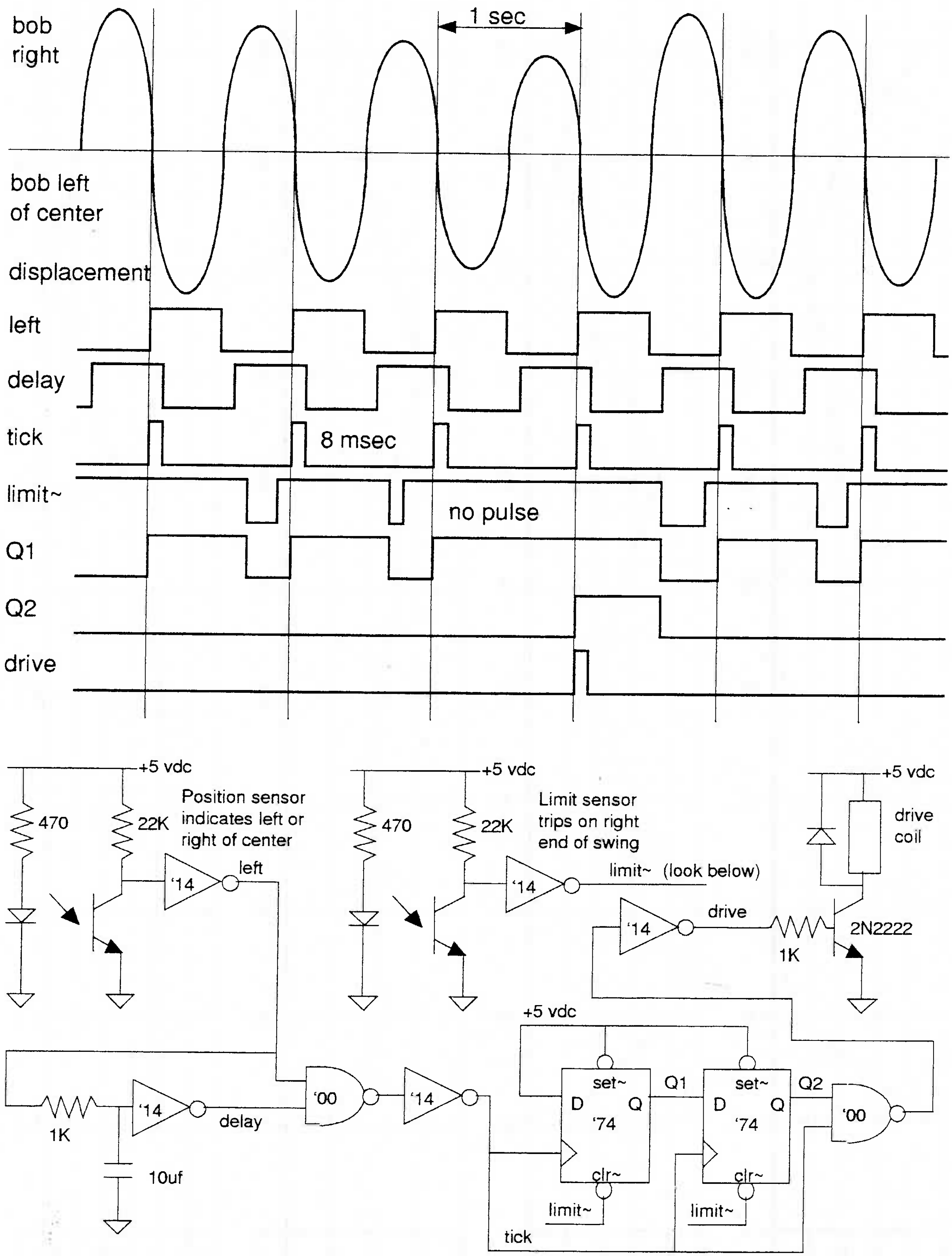


Figure 2 Drive electronics and timing

Fig. 4: (invar,0.001ss,const angle) (575hrs start) (l4b.g) (1.8T 15B) (day averages)

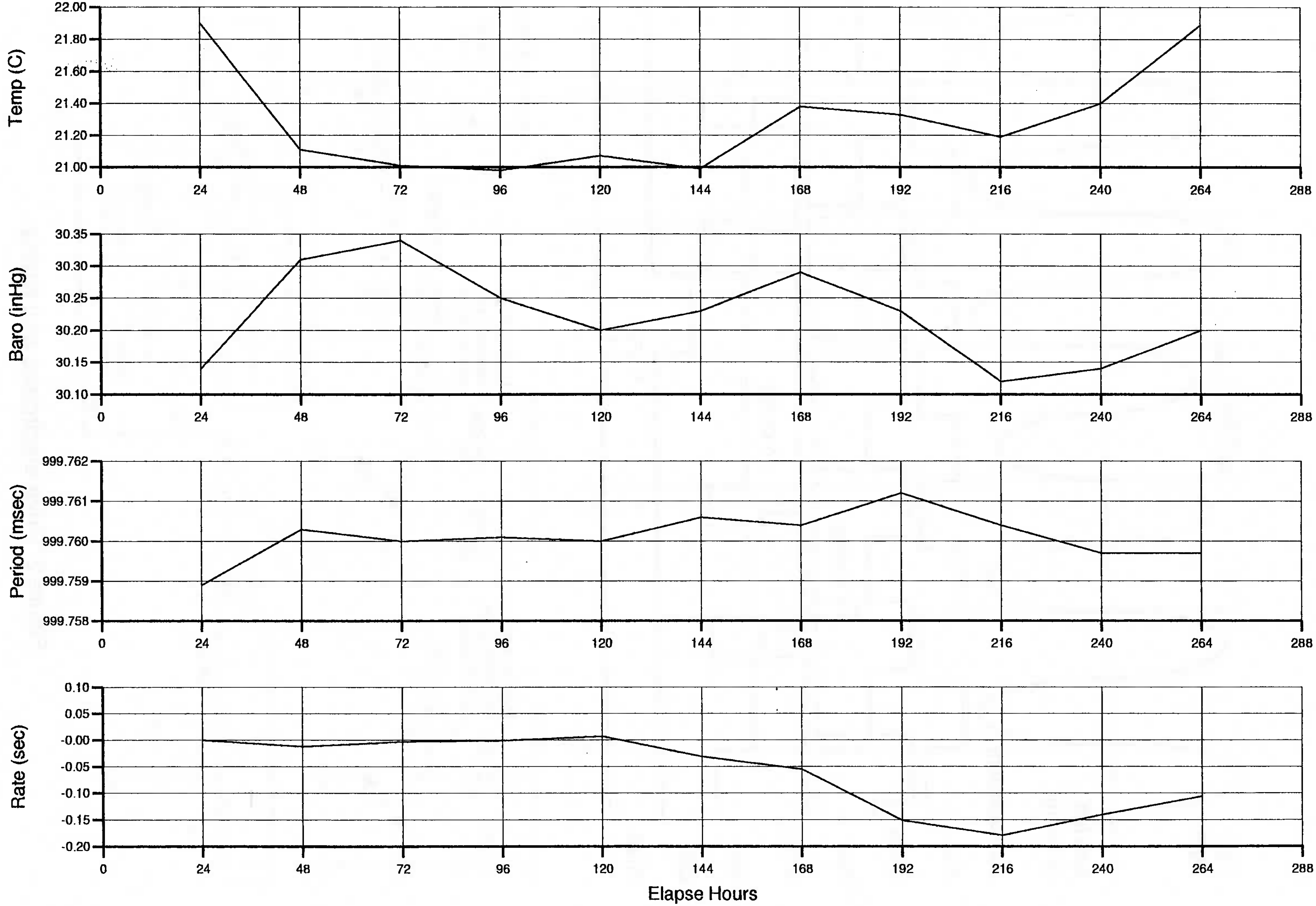


Fig. 5 (invar,0.001ss,const angle) (671 start) (p5a.g) (1.7T) (hr averages)

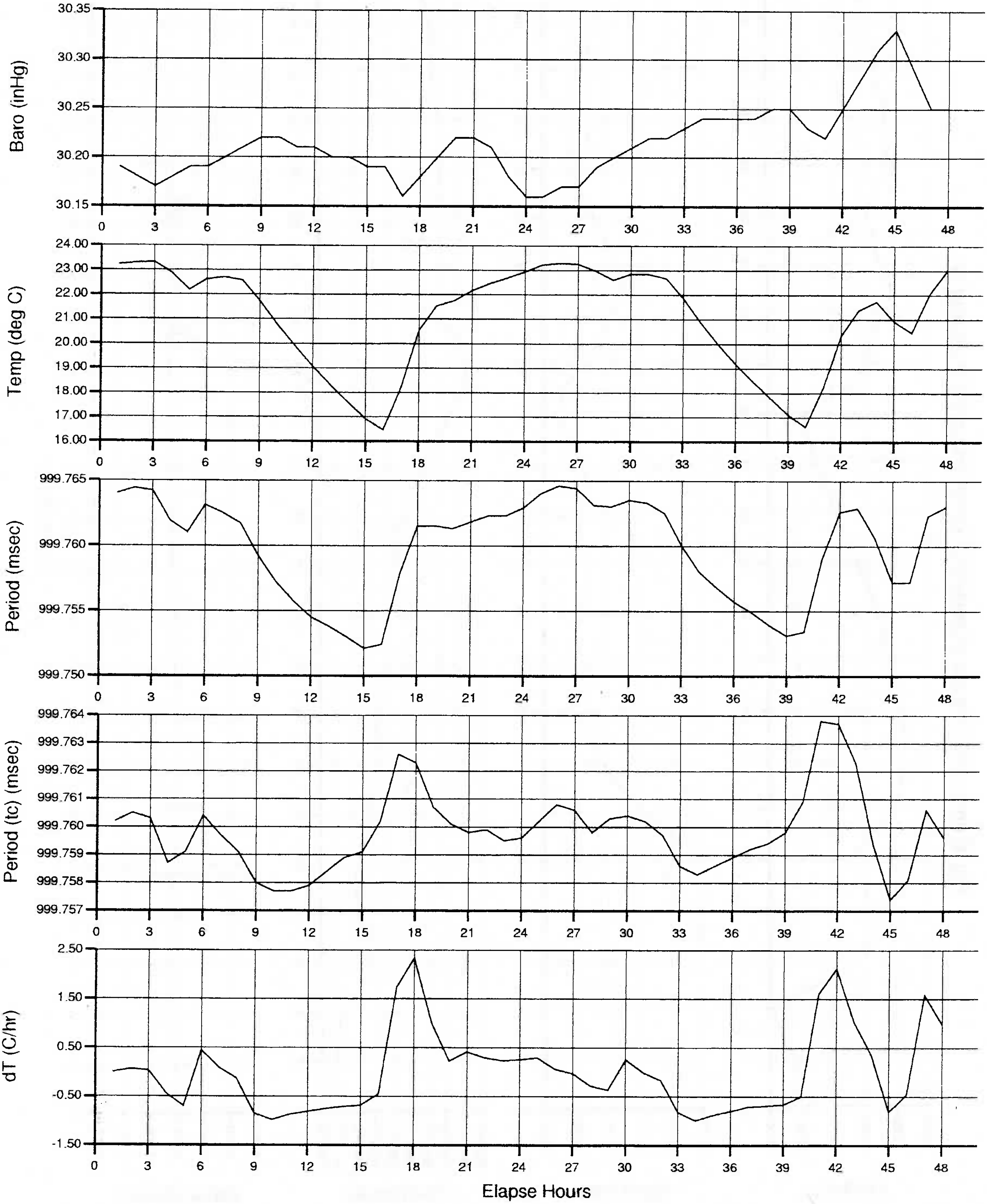
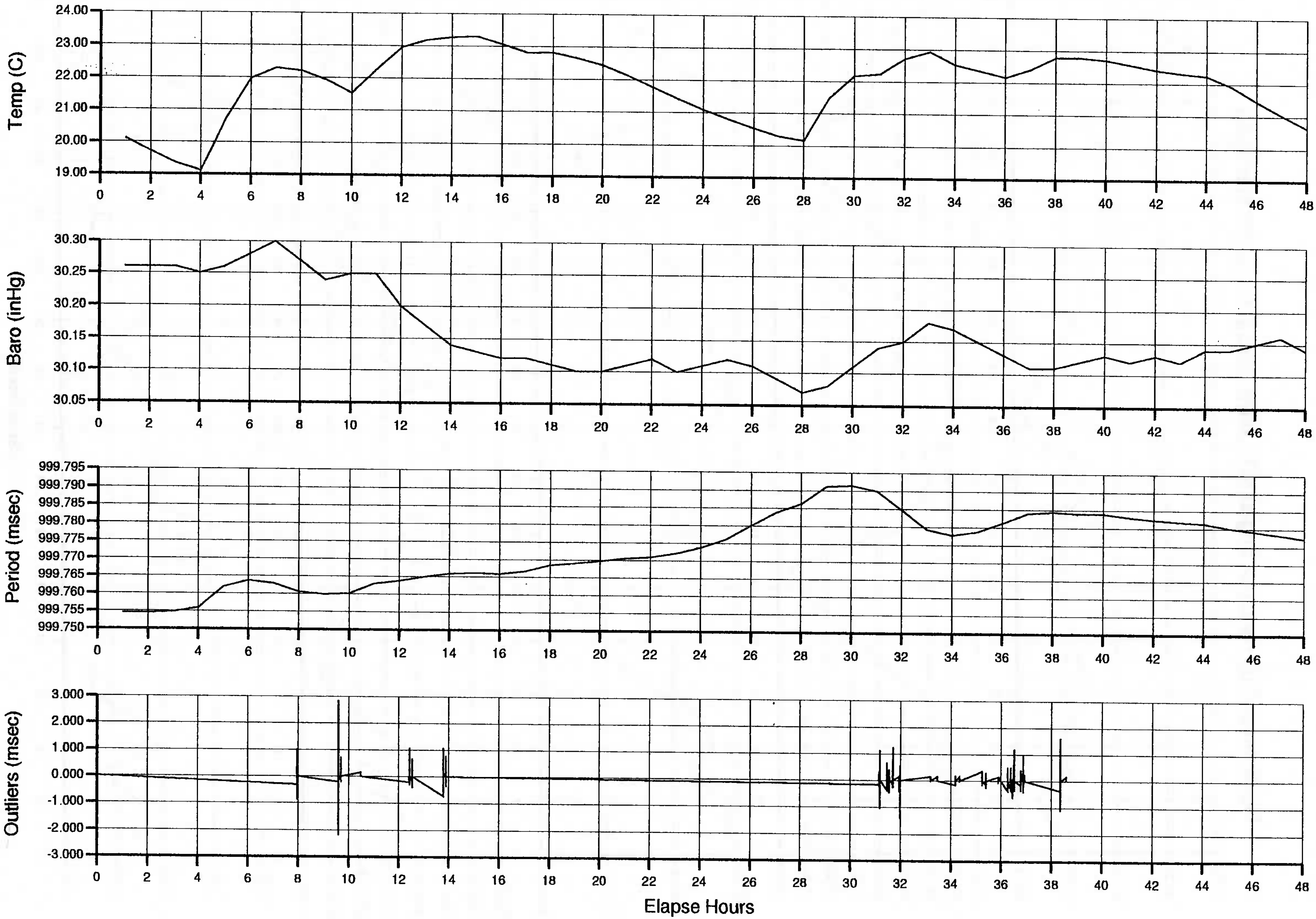


Fig. 6 (invar,0.001ss,const angle)(875 start)(l4c.g)(hour averages)



Component	CLE ($\times10^{-6}$/°C)	Tempco (ppm)
Copper bob	16.8	0.40
Aluminum tube	23.8	1.44
Steel tube	11.6	0.70
Poplar rod	1.7	1.09
Steel rod	11.6	7.43
Invar rod	1.4	0.90
Invar rod	4.2	2.70 (3x)

Date	Rod	Flex	Measured tempco (ppm/°C)	Computed tempco rod tube bob (ppm/°C)	Difference (ppm/°C)
1/15/99	Invar36	0.001 ss	1.8	-0.94	2.74
	3 x cle		1.8	0.85	0.95
12/8/98	Invar36	Silk cloth	3.0	-0.20	3.2
	3 x cle		3.0	0.85	2.15
8/11/98	Drill rod	Silk cloth	6.0	6.33	-0.33
8/2/98	Poplar	Silk cloth	-5.0	-0.01	-4.99
7/29/98	Poplar	0.002 ss	19	-0.01	19.01
7/8/98	Drill rod	0.002 ss	9	6.33	2.67

Notes:

0.001 ss flexure is 0.001 in thick stainless steel shim stock 3 mm wide (two springs)

0.002 ss flexure is 0.002 in thick stainless steel shim stock 6.5 mm wide (two springs)

silk cloth is 9.5 mm wide scrap of silk fabric

poplar is 1/8th inch poplar dowel stock

flexure is 2 mm high

only the 1/15/99 tests have an aluminum tube compensator 30 mm long all others have steel

Figure 7. Tempcos of various pendulum configurations.

Fig 8 (invar,0.001ss,const angle) (12/13/98 start) (l4a.g)