Measuring a Clock's Temperature Response

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Every clock has some response to temperature change. Before we can compensate for temperature variation we need to be able to measure actual thermal response. If a clock changes period in direct linear proportion to temperature change we can use a single number to quantify thermal response. This number would be expressed in, for example, milliseconds (ms.) per degree centigrade. Even if the response is not linear for a very wide temperature range, a linear model may be a good approximation for a narrow temperature range. In this paper I will use data for ambient temperature and pendulum period taken for each pendulum cycle over an entire day to find an approximate thermal coefficient for a specific electro-magnetically driven clock. Since clocks have a certain thermal mass, period response lags ambient temperature changes. I also show how to model this effect.

Data comes from my second clock built in the 1970s and recently upgraded. Operation is nearly identical to clock #7 described in HSN 1999-3. The flexure is stainless steel, the rod is wood, and the bob is stainless steel. This is a 120 beat system with each full cycle of the pendulum taking about 1 second. Actually the clock is 2 minutes a day slow. Data runs from a little before midnight for 24 hours on February 10, 2000. Barometer variation during that day is small (0.07 inch Mercury.) Statistical tests do not support any barometric effect this day so we can concentrate only on temperature.

The easiest way to cycle temperature for me is to just use household ambient variations. Data starts during a nighttime cooling period. At 5:30 am the furnace come on. The furnace is off part of midday, then comes on again, and is finally shutoff for the night about 9 pm. Total temperature cycle is just less than 5 degrees centigrade. Data is averaged over 900 seconds, one-quarter hour.

Figure 1 shows the period (bold line) vs. ambient temperature (fine line.) Period is in milliseconds. There are a total of 93 data points. Notice that during quick temperature changes the period lags temperature by quite a bit. But temperature and period are closely related.

Figure 2 shows an estimate of period (bold line) based only on ambient temperature. Measured period (fine line) is also shown. This estimate is simply: period estimate equals temperature times some coefficient plus some offset:

$$P_e = aT_a + b.$$

A spreadsheet (Excel) is used for all calculation and plotting. I compute a value for 'a' and 'b' using function 'linest.' 'a' is 0.023 ms/°C. ('b' is just an offset and isn't important here.) This is a huge temperature effect of about 2 seconds per day per degree C. A wooden rod and stainless steel bob have an expected temperature coefficient of well less than 0.005 ms/°C so there are other effects.

'Linest' fits the best possible straight line through all 93 data points and returns the slope 'a.' 'Best' means that the sum of squares of the differences between sample data and estimated data is as small as possible. Linest also indicates 'goodness' of fit. Here this value is 0.93 with 1.00 being a perfect match. Figure 2 also shows actual error at each point. This error, in ms., is shown hatched.

Clearly heat doesn't flow into the system instantaneously; it takes some time. The temperature inside the system lags the temperature outside the system. Heat flow depends on temperature difference and on the 'resistance' of the system to heat flow. So the T_a used in the estimating formula above should really be the temperature of the system itself – T_s.

While it must be true that each clock part has a different thermal response, I am going to lump these together for now into a single 'average' value. Heat transfer theory indicates that T_s will lag T_a. Any given T_s is computed by taking an exponentially decaying weighted average of the current and 7 previous T_as. Weighting factors that compute this average are shown in figure 4. More points could be included in the average, but 8 (2 hours) are enough for now. This kind of average is a low pass filter. Slow change passes right through while quick ones are attenuated.

Weighting factors are chosen using an exponential function of a single number – 'r'. A small value gives a very quick decay and corresponds to low thermal resistance. A larger number represents slow system thermal response. Figure 3 is a plot of 'goodness' of fit for P_e using T_ss for various 'r' values. The best fit occurs when T_s is filtered with an 'r' value of just less than 4. Figure 4 show the weighting factors for r = 3.8. In this case about 61% of thermal effect takes effect in the first 3/4 hour. (.26+.20+.15+ = .61) See appendix for much more on thermal lag spreadsheet modeling.

Finally figure 5 is like figure 2 except that estimated period is derived from T_s instead of T_a. The error is much smaller and the fit better (0.985.) Because of the lag, T_s is attenuated from T_a and has a smaller range (0.16°C less), now the temperature coefficient is 0.025 ms/°C. (Same period variation in a smaller temperature range.)

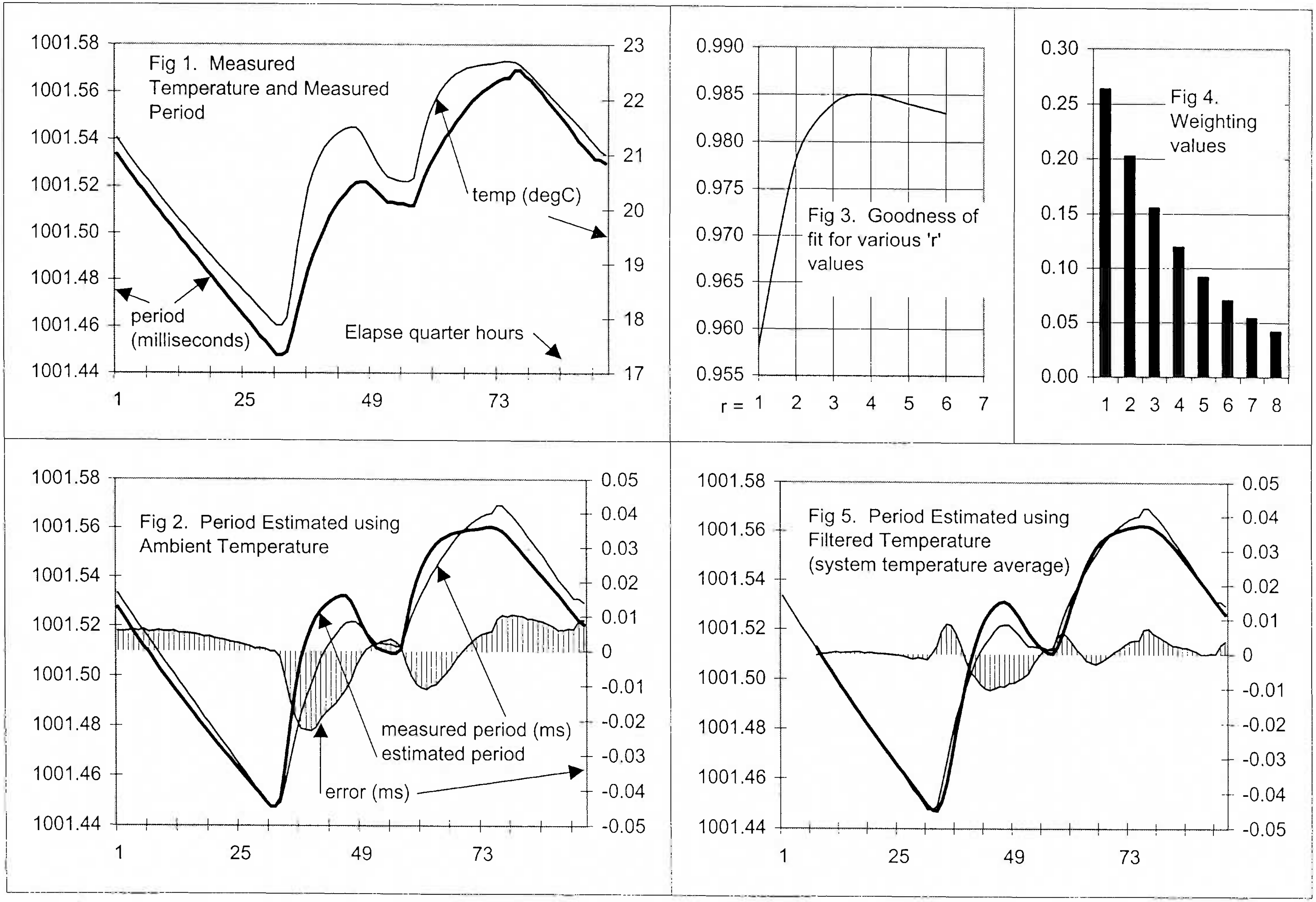


Fig 1. Measured Temperature and Measured Period
1001.58
1001.56
1001.54
1001.52
1001.50
1001.48
1001.46
1001.44
23
22
21
20
19
18
17
1
25
49
73
temp (degC)
period (milliseconds)
Elapse quarter hours
Fig 3. Goodness of fit for various 'r' values
0.990
0.985
0.980
0.975
0.970
0.965
0.960
0.955
r = 1 2 3 4 5 6 7
Fig 4. Weighting values
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1 2 3 4 5 6 7 8
Fig 2. Period Estimated using Ambient Temperature
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
measured period (ms)
estimated period
error (ms)
Fig 5. Period Estimated using Filtered Temperature (system temperature average)

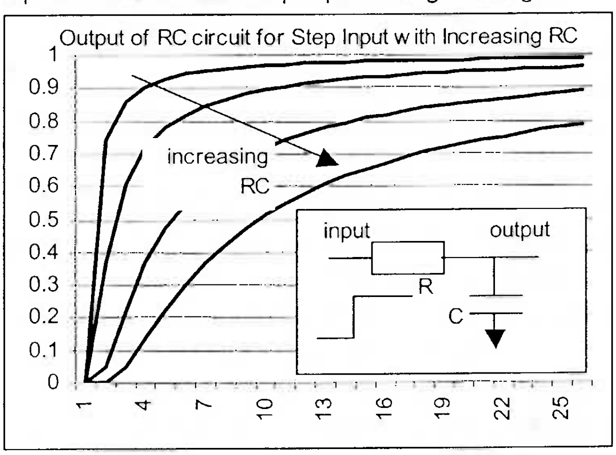
Appendix: Thermal Lag with a Spreadsheet

This appendix shows how to use a spreadsheet to approximate thermal lag in a clock. This is not easy going, nor is the model perfect but I think it is a useful, if limited, tool.

Heat transfer for unsteady-state systems is a complex area. I'm not going to go very deep here. Usually an 'infinite plate' example is used. A uniformly hot, infinite steel plate is suddenly exposed, at its one surface, to cool air. What is the temperature profile in the plate as time passes? The solution to the differential equation has exponential shape. Our problem is much more complex. What is the temperature at all points in say a pendulum (flexure, rod, bob, etc) with change in ambient temperature? The answer depends upon the temperature difference and properties of air and pendulum materials.

When ambient temperature changes, clock temperature starts to change quickly but then the rate of change slows exponentially. This exponential change after ambient change is exactly the same as the change in output voltage on a resistor/capacitor circuit from input voltage change.

The graph (below) shows how voltage on a capacitor varies with step input voltage change.



Voltage and temperature are analogous. The greater the product of resistance times capacitance (the time constant), the longer it takes for the output to equal the input. In a pendulum, a metal rod will have a shorter time constant than a wooden one because heat flows more easily into metal than wood.

The equation:

$$V_{output}(t) = \frac{1}{RC}\int_{-\infty}^{t} V_{input}(x)e^{\frac{-(t-x)}{RC}}dx$$

shows how to compute the voltage on a capacitor given the history of voltages input to the RC circuit. (V-voltage as a function of t-time. X is a dummy variable for integration from the beginning of time to time t.

The factor $e^{\frac{-(t-x)}{RC}}$ is 1 at $x = t$ but gets smaller quickly as $x \to -\infty$. In the case of a pendulum it gets so small in at most an hour or two that we can simply let it be zero. The net effect in this equation of factor 1/RC and the integral is a 'weighted' average of input voltage. R, C, and V have units chosen to be compatible. They are really just scale factors.

The effective temperature of the pendulum as a function of air temperature history obtained by sampled data taken by computer at fixed intervals can be numerically integrated to approximate the equation above. The number of points used 'n' is a compromise between accuracy, difficulty, system noise, and approximate thermal lag.

$$T_p[i] = \sum_{0}^{n} T_s[i-n]W_n \, .$$

$T_p[i]$ is the effective pendulum temperature at time i; $T_s[i-n]$ is ambient sensor temperature at time $[i-n]$. W_n are exponential weighting factors based on the thermal response 'r' (a term analogous to the RC product above) of the pendulum The sum of n+1 weighting factors adds up to 1. On a spreadsheet this looks like:

r =	2			
n	0	1	2	sum(
-n/r	0	-0.5	-1	exp(-n/r)):
exp(-n/r)	1	0.606531	0.367879	1.9744101
Wn	0.50648	0.307196	0.186324	sum(Wn)=1

W[n] = [exp(-n/r)]/[sum exp(-n/r)]

Now, finally, to get pendulum temperature data from ambient (sensor) temperature data:

Ts	Tp r=2	Tp r=1	Tp r=4
19			
19			
19	19.000	19.000	19.000
20	19.506	19.665	19.419
20	19.814	19.910	19.746
20	20.000	20.000	20.000
20	20.000	20.000	20.000

The formula for 'Tp r=2' is for say the 4^{th} row: =A4*0.50648+A3*0.307196+A2*0.186324

Note above how much slower Tp approaches 20 degrees as r increases. Also notice that Tp starts with the third Ts for n=2. In the paper n=8.

In the paper the entire pendulum system was 'lumped' together which is only a first approximation. Individual terms for flexure length, rod length, bob length, bob buoyancy, flexure spring constant, and mounting stiffness must be factors but experimental data doesn't allow these to be separated as yet.

The temperature sensor's time constant is not specified by the manufacturer because it is too application dependent. In this paper, the clock's response is slower than the sensor's.